Solutions - Homework 1

(Due date: September 23rd @ 11:59 pm)

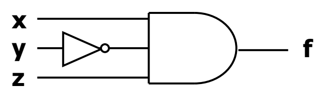
Presentation and clarity are very important! Show your procedure!

PROBLEM 1 (31 PTS)

a) Simplify the following functions using ONLY Boolean Algebra Theorems. For each resulting simplified function, sketch the logic circuit using AND, OR, XOR, and NOT gates. (15 pts)

✓ $F = \overline{\bar{x} + x(y+\bar{z})}$ ✓ $F(x,y,z) = \prod(M_2, M_4, M_6, M_7)$ ✓ $F = (z+\bar{y})(\bar{z}+x)(\bar{y}+x)$

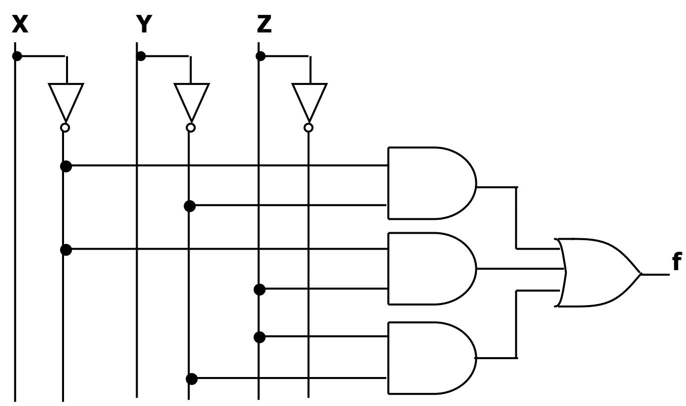
✓ $F = \overline{x(y+\bar{z})+\bar{x}} = \overline{x(y+\bar{z})}.x = (\bar{x}+\overline{y+\bar{z}}).x = (\overline{y+\bar{z}}).x = x\bar{y}z$



✓ $F(X,Y,Z) = \prod(M_2, M_4, M_6, M_7) = \sum(m_0, m_1, m_3, m_5) = \bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z + \bar{X}YZ + X\bar{Y}Z = \bar{X}\bar{Y} + \bar{X}YZ + X\bar{Y}Z$

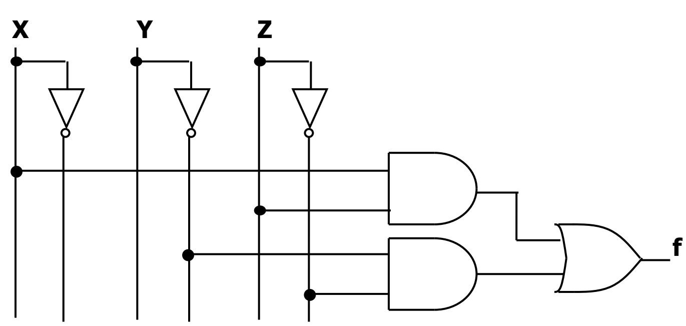
$F(X,Y,Z) = \bar{X}\bar{Y} + \bar{X}YZ + X\bar{Y}Z = \bar{X}(\bar{Y}+YZ) + X\bar{Y}Z = \bar{X}(\bar{Y}+Z) + X\bar{Y}Z = \bar{X}\bar{Y} + \bar{X}Z + X\bar{Y}Z$

$F(X,Y,Z) = \bar{X}\bar{Y} + \bar{X}Z + X\bar{Y}Z = \bar{X}\bar{Y} + Z(\bar{X}+X\bar{Y}) = \bar{X}\bar{Y} + Z(\bar{X}+\bar{Y}) = \bar{X}\bar{Y} + Z\bar{X} + Z\bar{Y}$



✓ $F = (Z+\bar{Y})(\bar{Z}+X)(\bar{Y}+X) = (Z+\bar{Y})(\bar{Z}+X)$ (Consensus Theorem)

$(Z+\bar{Y})(\bar{Z}+X) = ZX + \bar{Z}\bar{Y} + X\bar{Y} = ZX + \bar{Z}\bar{Y}$ (Consensus Theorem)



b) Using Boolean Algebra Theorems, prove that: $x(y \oplus z) = (xy) \oplus (xz)$ (6 pts)

$x(y \oplus z) = x(\bar{y}z + y\bar{z}) = x\bar{y}z + xy\bar{z}$

$(xy) \oplus (xz) = \overline{xy}xz + xy\overline{xz} = (\bar{x}+\bar{y})xz + xy(\bar{x}+\bar{z}) = \bar{x}xz + \bar{y}xz + xy\bar{x} + xy\bar{z} = x\bar{y}z + xy\bar{z}$

$\therefore\ x(y \oplus z) = (xy) \oplus (xz)$

c) For the following Truth table with two outputs: (10 pts)
- Provide the Boolean functions using the Canonical Sum of Products (SOP), and Product of Sums (POS). (4 pts)
- Express the Boolean functions using the minterms and maxterms representations.
- Sketch the logic circuits as Canonical Sum of Products and Product of Sums. (4 pts)

x	y	z	f_1	f_2
0	0	0	1	1
0	0	1	0	0
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	0	1
1	1	0	1	1
1	1	1	0	1

Sum of Products

$f_1 = \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} + \bar{X}YZ + X\bar{Y}\bar{Z} + XY\bar{Z}$

$f_2 = \bar{X}\bar{Y}\bar{Z} + \bar{X}YZ + X\bar{Y}Z + XY\bar{Z} + XYZ$

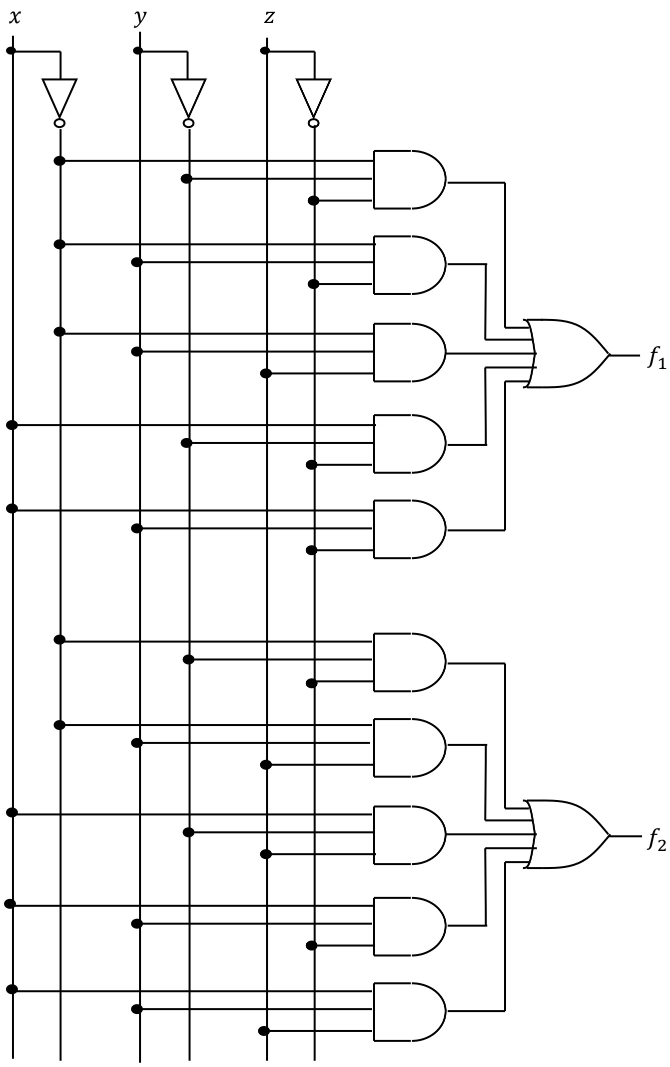
Product of Sums

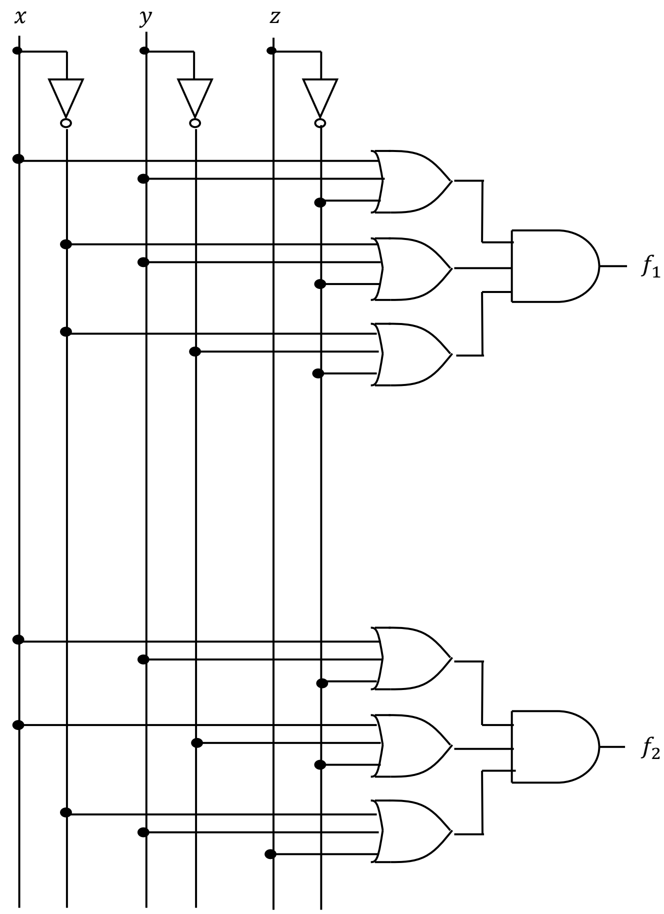
$f_1 = (X + Y + \bar{Z})(\bar{X} + Y + \bar{Z})(\bar{X} + \bar{Y} + \bar{Z})$

$f_2 = (X + Y + \bar{Z})(X + \bar{Y} + Z)(\bar{X} + Y + Z)$

Minterms and maxterms: $f_1 = \sum(m_0, m_2, m_3, m_4, m_6) = \prod(M_1, M_5, M_7)$.

$f_2 = \sum(m_0, m_3, m_5, m_6, m_7) = \prod(M_1, M_2, M_4)$.

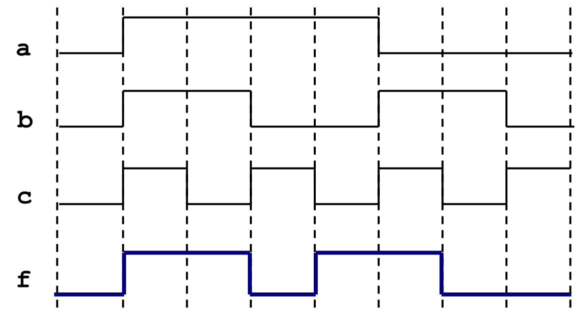




PROBLEM 2 (24 PTS)

a) The following is the timing diagram of a logic circuit with 3 inputs. Sketch the logic circuit that generates this waveform. Then, complete the VHDL code. (8 pts)

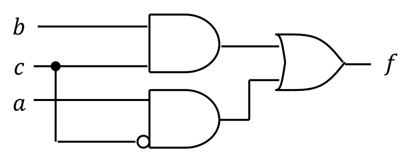
```
library ieee;
use ieee.std_logic_1164.all;

entity wave is
  port ( a, b, c: in std_logic;
             f: out std_logic);
end wave;

architecture struct of wave is
   signal x: std_logic;
begin
   x <= not(C) and a;
   f <= (b and c) or x;

end struct;
```



a	b	c	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

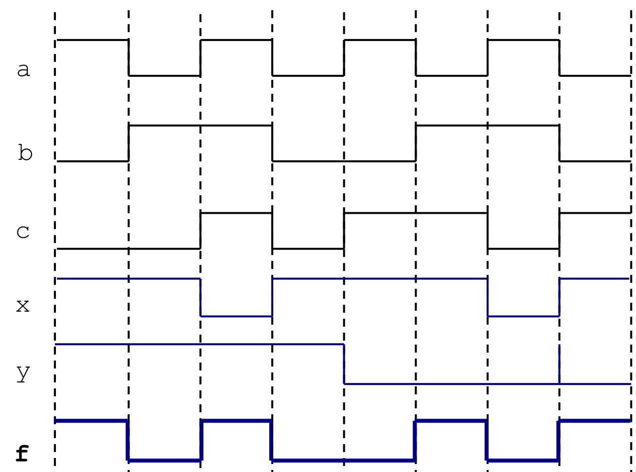
c \ ab	00	01	11	10
0	0	0	1	1
1	0	1	1	0

$f = bc + a\bar{c}$

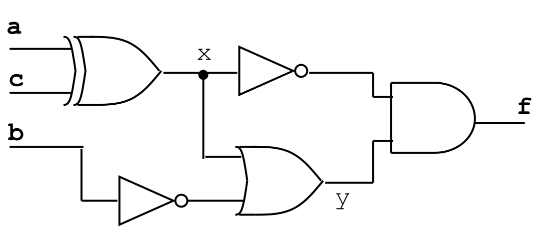


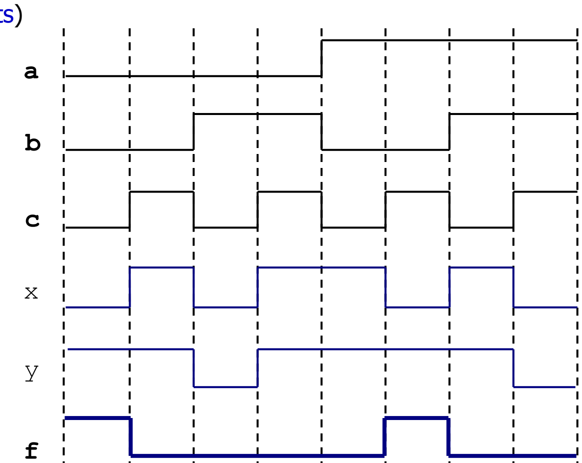
b) Complete the timing diagram of the logic circuit whose VHDL description is shown below: (5 pts)

```
library ieee;
use ieee.std_logic_1164.all;

entity circ is
  port ( a, b, c: in std_logic;
          f: out std_logic);
end circ;

architecture struct of circ is
  signal x, y: std_logic;

begin
  f <= y xor (not a);
  x <= a nand b;
  y <= x xnor (not c);
end struct;
```

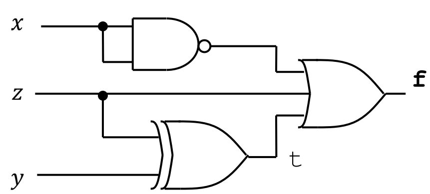


c) Complete the timing diagram of the following circuit: (5 pts)





d) Complete the truth table describing the output of the following circuit and write the simplified Boolean equation (6 pts).



x	y	z	t	f
0	0	0	0	1
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	0	1

$f = \bar{x} + y + z$

PROBLEM 3 (10 PTS)

- Complete the truth table for a circuit with 4 inputs x, y, z, w that activates an output ($f = 1$) when the number of 1's in the inputs is even. For example: If $xyzw = 1100 \rightarrow f = 1$. If $xyzw = 1011 \rightarrow f = 0$.
- Design (provide the simplified Boolean equation for f and sketch the logic circuit).

x	y	z	w	f
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

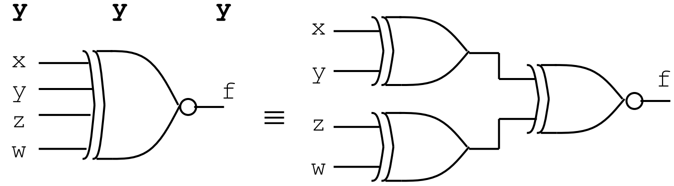
zw \ xy	00	01	11	10	
00	1	0	1	0	$\bar{z}\bar{w}$
01	0	1	0	1	$\bar{z}w$
11	1	0	1	0	zw
10	0	1	0	1	$z\bar{w}$
	$\bar{x}\bar{y}$	$\bar{x}y$	xy	$x\bar{y}$	

$$f = \bar{x}\bar{y}\bar{z}\bar{w} + \bar{x}\bar{y}zw + \bar{x}y\bar{z}w + \bar{x}yz\bar{w} + xy\bar{z}\bar{w} + xyzw + x\bar{y}\bar{z}w + x\bar{y}z\bar{w}$$

$$f = \bar{x}\bar{y}(\overline{z \oplus w}) + \bar{x}y(z \oplus w) + xy(\overline{z \oplus w}) + x\bar{y}(z \oplus w)$$

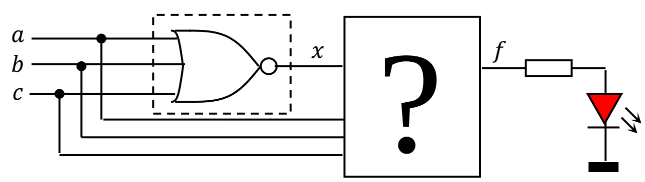
$$f = (\overline{x \oplus y})(\overline{z \oplus w}) + (x \oplus y)(z \oplus w)$$

$$f = \overline{(x \oplus y) \oplus (z \oplus w)} = \overline{(x \oplus y \oplus z \oplus w)}$$



PROBLEM 4 (11 PTS)

- Design a circuit (simplify your circuit) that verifies the logical operation of a 3-input NOR gate. f = '1' (LED ON) if the NOR gate does NOT work properly. Assumption: when the NOR gate is not working, it generates 1's instead of 0's and vice versa.



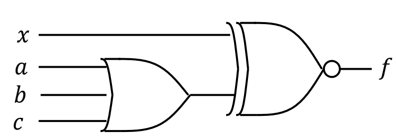
x	a	b	c	f	x_{good}
0	0	0	0	1	1
0	0	0	1	0	0
0	0	1	0	0	0
0	0	1	1	0	0
0	1	0	0	0	0
0	1	0	1	0	0
0	1	1	0	0	0
0	1	1	1	0	0
1	0	0	0	0	1
1	0	0	1	1	0
1	0	1	0	1	0
1	0	1	1	1	0
1	1	0	0	1	0
1	1	0	1	1	0
1	1	1	0	1	0
1	1	1	1	1	0

bc \ xa	00	01	11	10	
00	1	0	1	0	$\bar{b}\bar{c}$
01	0	0	1	1	$\bar{b}c$
11	0	0	1	1	bc
10	0	0	1	1	$b\bar{c}$
	$\bar{x}\bar{a}$	$\bar{x}a$	xa	$x\bar{a}$	

$$f = xa + cx + bx + \bar{x}\bar{a}\bar{b}\bar{c}$$

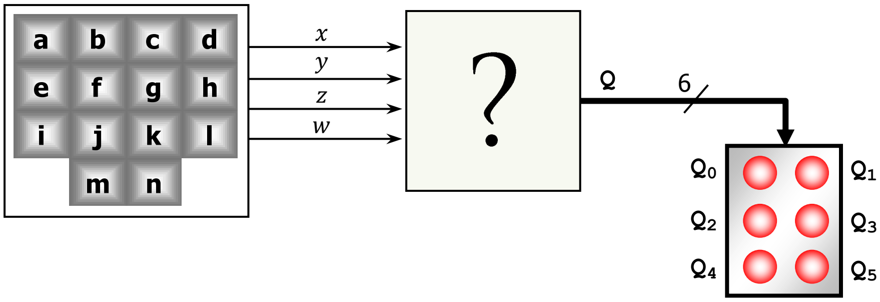
$$f = x(a + b + c) + \bar{x}(\overline{a + b + c})$$

$$f = \overline{x \oplus (a + b + c)}$$

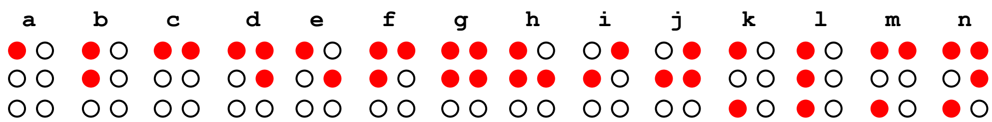


PROBLEM 5 (25 PTS)

- A 14-letter keypad produces a 4-bit code as shown in the table. We want to design a logic circuit that converts those 4-bit codes to Braille code, where the 6 dots are represented by LEDs. A raised (or embossed) dot is represented by an LED ON (logic value of '1'). A missing dot is represented by a LED off (logic value of '0').
  - ✓ Complete the truth table for each output (Q_0-Q_5). (4 pts)
  - ✓ Provide the simplified expression for each output (Q_0-Q_5). Use Karnaugh maps for Q_3, Q_2, Q_0 and the Quine-McCluskey algorithm for Q_5, Q_4, Q_1. Note it is safe to assume that the codes `1110` and `1111` will not be produced by the keypad.



x	y	z	w	Letter
0	0	0	0	a
0	0	0	1	b
0	0	1	0	c
0	0	1	1	d
0	1	0	0	e
0	1	0	1	f
0	1	1	0	g
0	1	1	1	h
1	0	0	0	i
1	0	0	1	j
1	0	1	0	k
1	0	1	1	l
1	1	0	0	m
1	1	0	1	n
1	1	1	0	
1	1	1	1	



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x	y	z	w	Q_5	Q_4	Q_3	Q_2	Q_1	Q_0	Letter
0	0	0	0	0	0	0	0	0	1	a
0	0	0	1	0	0	0	1	0	1	b
0	0	1	0	0	0	0	0	1	1	c
0	0	1	1	0	0	1	0	1	1	d
0	1	0	0	0	0	1	0	0	1	e
0	1	0	1	0	0	0	1	1	1	f
0	1	1	0	0	0	1	1	1	1	g
0	1	1	1	0	0	1	1	0	1	h
1	0	0	0	0	0	0	1	1	0	i
1	0	0	1	0	0	1	1	1	0	j
1	0	1	0	0	1	0	0	0	1	k
1	0	1	1	0	1	0	1	0	1	l
1	1	0	0	0	1	0	0	1	1	m
1	1	0	1	0	1	1	0	1	1	n
1	1	1	0	X	X	X	X	X	X	
1	1	1	1	X	X	X	X	X	X	

Q_3

zw \ xy	00	01	11	10
00	0	1	0	0
01	0	0	1	1
11	1	1	X	0
10	0	1	X	0

Q_2

zw \ xy	00	01	11	10
00	0	0	0	1
01	1	1	0	1
11	0	1	X	1
10	0	1	X	0

Q_0

zw \ xy	00	01	11	10
00	1	1	1	0
01	1	1	1	0
11	0	1	X	1
10	1	1	X	1

$Q_5 = 0$
$Q_4 = xz + xy$
$Q_3 = \bar{x}y\bar{w} + \bar{x}zw + x\bar{z}w$
$Q_2 = yz + \bar{x}\bar{z}w + x\bar{y}\bar{z} + x\bar{y}w$
$Q_1 = \bar{x}\bar{y}z + y\bar{z}w + \bar{x}z\bar{w} + x\bar{z}$
$Q_0 = \bar{x} + y + z$

- $Q_4 = \sum m(10,11,12,13) + \sum d(14,15)$.

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants	1-literal implicant
2	m_{10}= 1010 ✓ m_{12}= 1100 ✓	$m_{10,11}$ = 101- ✓ $m_{10,14}$ = 1-10 ✓ $m_{12,13}$ = 110- ✓ $m_{12,14}$ = 11-0 ✓	**$m_{10,11,14,15}$ = 1-1-** ~~$m_{10,14,11,15}$ = 1-1-~~ **$m_{12,13,14,15}$ = 11--** ~~$m_{12,14,13,15}$ = 11--~~	We can't combine any further, so we stop here
3	m_{11}= 1011 ✓ m_{13}= 1101 ✓ m_{14}= 1110 ✓	$m_{11,15}$ = 1-11 ✓ $m_{13,15}$ = 11-1 ✓ $m_{14,15}$ = 111- ✓		
4	m_{15}= 1111 ✓			

Prime Implicants		Minterms			
		10	11	12	13
$m_{10,11,14,15}$	xz	**X**	X		
$m_{12,13,14,15}$	xy			**X**	X

$\rightarrow Q_4 = xz + xy$

- $Q_1 = \sum m(2,3,5,6,8,9,12,13) + \sum d(14,15)$.

Number of ones	4-literal implicants	3-literal implicants	2-literal implicants	1-literal implicant
1	m_2 = 0010 ✓ m_8 = 1000 ✓	**$m_{2,3}$ = 001-** **$m_{2,6}$ = 0-10** $m_{8,9}$ = 100- ✓ $m_{8,12}$ = 1-00 ✓	**$m_{8,9,12,13}$ = 1-0-** ~~$m_{8,12,9,13}$ = 1-0-~~ **$m_{12,14,13,15}$ = 11--** ~~$m_{12,13,14,15}$ = 11--~~	We can't combine any further, so we stop here
2	m_3 = 0011 ✓ m_5 = 0101 ✓ m_6 = 0110 ✓ m_9 = 1001 ✓ m_{12}= 1100 ✓	**$m_{5,13}$ = -101** **$m_{6,14}$ = -110** $m_{9,13}$ = 1-01 ✓ $m_{12,13}$ = 110- ✓ $m_{12,14}$ = 11-0 ✓		
3	m_{13}= 1101 ✓ m_{14}= 1110 ✓	$m_{13,15}$ = 11-1 ✓ $m_{14,15}$ = 111- ✓		
4	m_{15}= 1111 ✓			

$Q_1 = \bar{x}\bar{y}z + \bar{x}z\bar{w} + y\bar{z}w + yz\bar{w} + x\bar{z} + xy$

Prime Implicants		Minterms							
		2	3	5	6	8	9	12	13
$m_{2,3}$	$\bar{x}\bar{y}z$	X	**X**						
$m_{2,6}$	$\bar{x}z\bar{w}$	X			**X**				
$m_{5,13}$	$y\bar{z}w$			**X**					X
$m_{6,14}$	$yz\bar{w}$				X				
$m_{8,9,12,13}$	$x\bar{z}$					**X**	**X**	**X**	X
$m_{12,14,13,15}$	xy							X	X

$\rightarrow Q_1 = \bar{x}\bar{y}z + y\bar{z}w + x\bar{z} + \bar{x}z\bar{w}$